Exercises

- Let k be your age in years. Find a set consisting of k consecutive composite integers.
- Redo the proof: that there are infinitely many primes of the form $4k+3$, but use

$$N = (p_1 p_2 ... p_n)^2 + 2.$$

- Prove that any prime of the form $3k+1$ is of the form $6k+1$ (Hints: consider the parity of k).
- Prove that any prime $p > 3$ is either of the form $6k+1$ or of the form $6k+5$ for some integer k.
- Prove that the product of any two integers of the form $6k+1$ is of that same form (e.g. $6k+1$).
- Prove that there are infinitely many primes of the form $6k+5$ (Hints: use some N in similar form).
- Prove that any positive integer of the form $6k+5$ must have some prime factor of the same form (Hints: consider its divisibility by $2, 3$).
- Show that if all three of p, $p+2$ and $p+4$ are prime, then the only possible choice is $p = 3$. (Hints: dividing p by 3.)
- Prove or disprove the following statements,

  (1) If $a^2 \mid b^3$, then $a \mid b$.
  (2) If $a^2 \mid b^2$, then $a \mid b$.
  (3) If $3 \mid a^2$ then $3 \mid a$.
  (4) If $3 \mid a^4$ then $3 \mid a$.
  (5) If $3 \mid a^3 b$ then $3 \mid a$ or $3 \mid b$.

- Let n and e be positive integers and p be a prime. Denote $p^e \parallel n$ if $p^e \mid n$ but $p^{e+1} \nmid n$.

  Given that $p^{e_1} \parallel m$ and $p^{e_2} \parallel n$ for some positive integers p, e_1, e_2, m, n.

  (1) What power of p exactly divides $m+n$? Prove it.
  (2) What power of p exactly divides mn? Prove it.
  (3) What power of p exactly divides m^n? Prove it.