Exercises

- Find some integer solutions to the following equation (if the solution exists) or show the the solution does not exist.

  (1) $2x + 3y = 4$
  (2) $15x + 51y = 41$
  (3) $121x - 88y = 572$

- Use the Euclidean algorithm to find the greatest common divisor of 780 and 150 and express it in terms of the two integers.

- Find the greatest common divisor d of $35, 55, 77$. Then find some integers x, y, z such that $35x + 55y + 77z = d$.

- Show that if a and b are positive integers where a is even and b is odd, then $\gcd(a, b) = \gcd(a/2, b)$.

- Show whether the following arguments are correct. If so prove it; otherwise, find a counterexample.

  (1) If $d = \gcd(a, b)$, then $\gcd(a/d, b) = 1$.
  (2) If $\gcd(a, c) = \gcd(b, c) = 1$ then $\gcd(ab, c) = 1$.
  (3) If $\gcd(a, b) = 1$ and $k > 0$, then $\gcd(ka, b) = k$.

- Redo the proof: that there are infinitely many primes of the form $4k + 3$, but use

$$N = (p_1 p_2 ... p_n)^2 + 2.$$

- Prove that there are infinitely many primes of the form $6k + 5$.