

Exercises: in your proof, write down the full details.

- (1) Let a, b, c and d be integers with a and c nonzero. Prove that if $a \mid b$ and $c \mid d$, then $ac \mid bd$.
- (2) Prove that if $ac \mid bc$, then $a \mid b$.
- (3) Is the following statement true?

If $a \mid b$ and $b \mid a$, then $a = b$.

- (4) Prove that the sum of two even integers is even, the sum of two odd integers is even and the sum of an even integer and an odd integer is odd.
- (5) Show that the product of two even integers is even, the product of two odd integers is odd and the product of an even integer and an odd integer is even.
- (6) Prove that the square of every odd integer is of the form $8m + 1$.
- (7) Let m be a positive integers. What values could $\gcd(m, m + 1)$ take?
- (8) Let m be a positive integers. What values could $\gcd(m, m + 2)$ take?
- (9) Prove that if m is a positive integer, then $3m + 2$ and $5m + 3$ are relatively prime.
- (10) Prove that $\gcd(m + 1, n + 1) \mid (mn - 1)$.
- (11) Prove that if m and n are integers where $\gcd(m, n) = 1$, then $\gcd(m + n, m - n) = 1$ or 2 . Note one needs to show both are achievable.