Exercises: in your proof, write down the full details.

(1) Let $n = 12$. Determine $ord_n(a)$ for all a in the complete reduced residue system modulo n.

(2) Let $n = 25$. Given that 2 is a primitive root modulo n, find all primitive roots modulo n.

(3) Show that if $\bar{a}$ is an inverse of a modulo n, then $ord_n(a) = ord_n(\bar{a})$.

(4) Let p and q be two distinct odd prime numbers. Prove that there is NO primitive root modulo pq. (Hints: use Euler's theorem).

(5) Review the proof for the theorem: If $\gcd(a, n) = 1$ with $n > 0$, the positive integer x is a solution the congruence $a^x \equiv 1 \pmod{n}$ if and only if $ord_n(a) \mid x$.

(6) Review the proof for the theorem: If $\gcd(a, n) = 1$ with $n > 0$, then

$$a^i \equiv a^j \pmod{n}$$

if and only if

$$i \equiv j \pmod{ord_n(a)}.$$

(7) Find all quadratic residues of 13.