Exercises

(1) Prove using mathematical induction that $n < 3^n$ for all positive integers n.

(2) Prove using mathematical induction that $2^n < n!$ for all integers $n \geq 4$.

(3) Prove using mathematical induction that

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

is true for all positive integers n.

(4) Prove using mathematical induction that

$$\sum_{i=1}^{n} (-1)^i i^2 = (-1)^n \frac{n(n+1)}{2}$$

for all positive integers n.

(5) Explain whether the following strong induction proof is correct or not (and give the reason).

Claim: for every positive number n, we have $n = 1$.

Proof. Base case is correct since if $n = 1$, then $n = 1$. Induction step follows as:

Assume that the claim holds for $n \leq k$ (this means $n = 1$ for all $n \leq k$). We prove that the claim also holds for $n = k + 1$ in the following way: we factor $k + 1 = a \cdot b$ for positive a, b. Then we use the (strong) induction step on a and b. We have $a = 1$ and $b = 1$ due to (strong) induction hypothesis. Therefore, $k + 1 = a \cdot b = 1 \cdot 1 = 1$.

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