

- (1) (3 marks) Which of the following are algebraic integers? Justify your answer.
- $1 + \frac{\sqrt{-3}}{2} + \frac{\sqrt{5}}{2}$
 - $\frac{4+\sqrt{7}}{2}$
 - $\frac{\sqrt{x}+\sqrt{y}}{2}$ where $x, y \equiv 3 \pmod{4}$
 - $\beta = 3/(1 - \alpha)$ where $\alpha \in \mathbb{C}$ is a root of the following monic irreducible polynomial: $f(x) = x^3 - 3x - 25$.
- (2) (2 marks) Let $K = \mathbb{Q}(\alpha)$ where $\alpha \in \mathbb{C}$ is a root of the following monic irreducible polynomial $f(x) = x^3 - 4x + 2$. Let $\beta = \alpha + \alpha^2$. Compute $Disc(\beta)$.
- (3) (3 marks) Let $K = \mathbb{Q}(\alpha)$ where $\alpha \in \mathbb{C}$ is a root of the following monic irreducible polynomial $f(x) = x^3 + 2x + 2$. Let $\beta = \alpha - \alpha^2$. Compute the field polynomial of β over K .
- (4) (2 marks) Let $K = \mathbb{Q}(\sqrt{2})$ and $I = \langle 7 + 2\sqrt{2} \rangle$ in \mathcal{O}_K .
- Find some $c \in I \cap \mathbb{Z}$.
 - Find another element $d \in I$ such that $Disc(c, d) \neq 0$.