(1) (3 marks) Let $R = \mathbb{Z} + \mathbb{Z}i$ be the ring of Gaussian integers.

- Prove that the units of R consist of $\{\pm 1, \pm i\}$.
- Let $S = \{a + bi \in R \mid b \equiv 0 \pmod 2\}$. Is S an ideal in R?
- Prove that $\langle 4 + i \rangle$ is a prime ideal; and 2 is not a prime in R.

(2) (3 marks) Let $R = \mathbb{Z} + \mathbb{Z}\sqrt{-5}$.

- Find all the units in R.
- Prove that $\langle 2, 1 + \sqrt{-5} \rangle = \langle 2, 1 - \sqrt{-5} \rangle$ in R.
- Prove that $\langle 3, 1 + \sqrt{-5} \rangle \neq \langle 3, 1 - \sqrt{-5} \rangle$ in R.

(3) (2 marks) Let I, J, K be ideals in a commutative ring R. Show the distributive law $I(J + K) = IJ + IK$.

(4) (2 marks) Two ideals I and J in a commutative ring R are called co-prime, if $I + J = R$. Prove that if I and J are co-prime, then $IJ = I \cap J$.